Section 5.1: Ratios, Rates, and Unit Rates Teacher Notes

POD:

Simplify the fractions

1) $\frac{9}{45}$ 1/5 2) $\frac{16}{64}$ 1/4 3) $\frac{45}{60}$ 3/4

Objective: To find ratios, rates, and unit rates

Vocabulary:

1.) Ratio - a comparison of 2 quantities (numbers) by division
2.) Rate - is a ratio of two quantities with different units (miles and hours)
3.) Unit Rate - A rate with the denominator of 1

How to Write a Ratio:

1.) Using a fraction bar $\frac{2}{3}$
2.) Using a colon 2 : 3
3.) Using the word "to" 2 to 3

All three ways are read the same "two to three"

Write ratios and find unit rates

1.) There are 12 boys and 20 girls in Mr. Luczak's math class. Write the ratio of boys to girls in three ways. 3/5 , 3 : 5, 3 to 5
2.) If you drive 183 miles in 3 hours. What is the unit rate (per hour rate)? 61 miles per hour

Finding a Rate from a Ratio Table.

Servings per package

Packages	3	6	9	12
Servings	13.5	27	40.5	54

Any of these: 13.5/3=4.5, 27/6=4.5, 40.6/9=4.5 or 54/12=4.5 servings per package

Cost Per Notebook

Notebooks	0	5	10	15
Cost (dollars)	0	9.45	18.90	28.35

Any of these: 9.45/5=$1.89, 18.90/10=$1.89, or 28.35/15=$1.89

Solving a Ratio Problem

You mix a ratio of $\frac{1}{4}$ cup of juice concentrate for every 2 cups of water. How much juice concentrate do you use to make 18 cups of juice?

HINT: Make a ratio table

Juice Concentrate	Water	Cups of Juice
$\frac{1}{4}$ (times 4)	2	$2\frac{1}{4}$
1 (times 2)	8	9
2	16	18

Section 5.2: Solving Proportions TEACHER NOTES (ONE DAY LESSON)

POD:

1.) If you bought 24 oranges for $6 what is the unit price per orange?

$0.25/orange

2.) If you earned $45 for 3 hours of work, what is the unit rate?

$15/hour

Objective: To test whether ratios form a proportion. To solve proportions using unit rates, mental math, and cross products.

Vocabulary:

1.) Proportion – an equation stating that two ratios are equal (i.e. $\frac{2}{4} = \frac{1}{2}$)

2.) Cross Products – Multiply diagonally to see if the proportions are equal.

We use can cross products to determine if two ratios form a proportion.

How to Determine if Two Ratios are a Proportion:

1.) Write the ratios as a proportion.

2.) Find the cross products. If they are:

a. **Equal** then they **are** a proportion

b. **Not Equal** then they are **not** a proportion

Examples:

Determine if each pair of rations form a proportion.

1.) $\frac{3}{8}, \frac{9}{24}$

$\frac{3}{8} \times \frac{9}{24}$ $\quad 3 \cdot 24 = 3 \cdot 9$

$72 = 72$

Yes, cross products are equal therefore they are a proportion.

2.) $\frac{8}{12}, \frac{6}{24}$

$\frac{8}{12} \times \frac{6}{24}$ $\quad 8 \cdot 24 = 12 \cdot 6$

$192 = 72$

No, cross products are not equal therefore they are not a proportion.

3.) You swim the first 4 laps in a swimming pool in 2.4 minutes. You complete 16 laps in 12 minutes. Is the number of laps proportional to your time?

$\frac{2.4}{4} = 0.6$ min/lap

$\frac{12}{16} = 0.75$ min/lap

No, it took you more time per lap when you did 16 laps.

4.) Tell whether x and y are proportional. Compare each ratio x to y in simplest form.

x	y
$\frac{1}{2}$	3
1	6
$\frac{3}{2}$	9
2	12

1/2/3 = 1/6

1/6 is 1/6

3/2/9 = 1/6

2/12 = 1/6

So yes, x and y are proportional

5.3 Writing and Solving Proportions Teacher Notes (ONE DAY LESSON)

POD:

Tell whether x and y are proportional. Show your work

x	1	2	3	4
y	7	8	9	10

Objective: To write and solve proportions using cross products.

Vocabulary:

1.) Proportion – an equation stating that two ratios are equal (i.e. $\frac{2}{4} = \frac{1}{2}$)

2.) Cross Products – Multiply diagonally to see if the proportions are equal.

We use can cross products to determine if two ratios form a proportion.

How to Solve a Proportion:

1.) Set up proportions (make sure the units correspond with one another)
2.) Multiply using cross products.
3.) Solve the equation by dividing.

Examples:

1.) $\frac{x}{8} = \frac{14}{4}$ $8 \cdot 14 = 4 \cdot x$ $\frac{112}{4} = \frac{4x}{4}$ $x = 28$	2.) $\frac{14}{6} = \frac{x}{15}$ $14 \cdot 15 = 6x$ $\frac{210}{6} = \frac{6x}{6}$ $x = 35$
3.) Write and Solve. How many points will a student need to earn on a test to get the given score. A test worth 80 points; a score of 95% $\frac{x}{80} = \frac{95}{100}$ $\frac{100x}{100} = \frac{7600}{100}$ $x = 76$ points	4.) Write and Solve. How many points will a student need to earn on a test to get the given score. A test worth 40 points; a score of 80% $\frac{x}{40} = \frac{80}{100}$ $\frac{100x}{100} = \frac{3200}{100}$ $x = 32$ points

Word Problems

5.) A recipe requires the ratio of sugar to flour to be 8 : 18.
A bowl of this recipe has 16 cups of sugar. How many cups of flour are in the bowl?

$\frac{8}{18} = \frac{16}{x}$

$\frac{8x}{8} = \frac{288}{8}$ $x = 36$

There are 36 cups of flour in the bowl (it's a big bowl☺)

6.) The ratio of water to oil in a muffin recipe is 3:4. The recipe uses 21 ounces of liquid (water and oil). How many ounces of the mix is water and how many ounces are oil?

Water:

$\frac{3}{7} = \frac{x}{21}$

$\frac{7x}{7} = \frac{63}{7}$

x = 9 ounces

Oil:

$\frac{4}{7} = \frac{x}{21}$

$\frac{7x}{7} = \frac{84}{7}$

x = 12 ounces

9 ounces of water and 12 ounces of oil

5.4 Complex Proportions and Graphs TEACHER NOTES

POD:

1.) $\frac{x}{8}=\frac{14}{4}$

x = 2.8

2.) $\frac{40}{50}=\frac{x}{80}$

x = 64

Objective: To use ratios and cross products to solve complex proportions and interpret graphs.

Steps for solving proportions:

1. Set up the proportions.
2. Solve using cross products.
3. Make sure to distribute if necessary!!

Examples: Solve each proportion.

1.) Macy has a recipe that requires 5 eggs, $\frac{3}{4}$ cup of flour, and $\frac{1}{2}$ cup of pudding. If Macy only has 3 eggs, how much flour will she need?

$\frac{3}{5}=\frac{x}{3/4}$

$5(x) = 3(\frac{3}{4})$

$\frac{5x}{5} = \frac{2.25}{5}$

$x = \frac{9}{20}$ cup of flour

2.) $\frac{3}{16}=\frac{24}{5x+1}$

$3(5x + 1) = 24 \cdot 16$

$15x + 3 = 384$

$-3 \quad -3$

$\frac{15x}{15} = \frac{381}{15}$

x = 25.4

3.) $\frac{5}{3x}=\frac{2}{20}$

$2(3x) = 5 \cdot 20$

$\frac{6x}{6} = \frac{100}{6}$

$x = 16.7$

4.) $\frac{x+3}{14}=\frac{20}{5}$

$5(x + 3) = 14 \cdot 20$

$5x + 15 = 280$

$-15 \quad -15$

$\frac{5x}{5} = \frac{265}{5}$

$x = 53$

5.) $\frac{2x-5}{18}=\frac{22}{15}$

$15(2x - 5) = 18 \cdot 22$

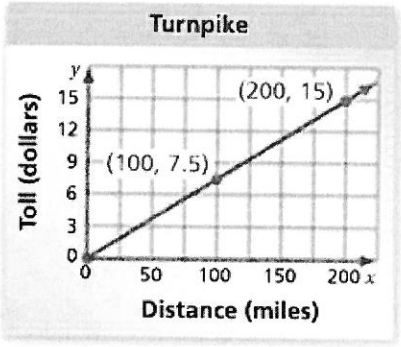
$30x - 75 = 396$

$+75 \quad +75$

$\frac{30x}{30} = \frac{471}{30}$

x = 15.7

The graph shows the toll y due on a turnpike for driving x miles. Your toll is \$7.50. How many *kilometers* did you drive?



The point (100, 7.5) on the graph shows that the toll is \$7.50 for driving 100 miles. Convert 100 miles to kilometers.

Method 1: Convert using a ratio.

1 mi ≈ 1.61 km

$$100 \text{ mi} \times \frac{1.61 \text{ km}}{1 \text{ mi}} = 161 \text{ km}$$

∴ So, you drove about 161 kilometers.

Method 2: Convert using a proportion.

Let x be the number of kilometers equivalent to 100 miles.

kilometers → $\frac{1.61}{1} = \frac{x}{100}$ ← kilometers

miles → miles

Write a proportion. Use 1.61 km ≈ 1 mi.

$1.61 \cdot 100 = 1 \cdot x$ Cross Products Property

5.5 Constant of Proportionality TEACHER NOTES

POD: Solve.

1.) $\frac{2x}{5} = \frac{6}{8}$

x = 1.875

2.) $\frac{x+1}{4} = \frac{4}{8}$

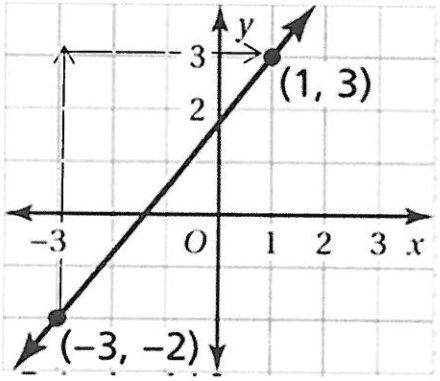
x = 1

Objective: Students will be able to determine if a relationship is a proportion. Students will be able to determine and explain the constant of proportionality from a relationship and identify slopes.

Vocabulary

1.) **Constant of Proportionality** - the **Unit Rate** of a set of data (constant = k), also known as the slope of a line
2.) **Constant of Proportionality Equation**: y = kx, where k = constant, (x, y) are coordinates on the graph
3.) **Slope**: the rate of change between any two points on a line. Slope = $\frac{\text{change in y}}{\text{Change in x}}$

Finding Slopes:



Slope = $\frac{\text{change in y}}{\text{change in x}}$

y went UP 5 (positive) and x went RIGHT 4 (positive) Therefore, the slope is 5/4 (KEEP as an improper Fraction because it tells you directions to move)

How to Determine if a Relationship is Proportional from a TABLE:

1.) Find the constant of proportionality - Unit Rate!!! Remember your units!
2.) If the constant IS the same for each value in the table, then it IS a proportional relationship
3.) If the constant is NOT the same for each value in the table, then it is NOT a proportional relationship.

Examples

Determine if each table represents a proportional relationship. If so, determine the constant of proportionality.

1.) Below is a table for the price of books at a local bookstore.

Books	$	Constant
1	3	3
3	9	3
4	12	3
7	18	2.57

Is the relationship proportional?

No, the relationship is not proportional because the constant is not the same for 18 books.

2.) Below is a table for the number of seed packets to make a plant.

Number of Seed Packets	Total Number of Plants	Constant
1	10	10
2	20	10
3	30	10
4	40	10

1a.) Is the relationship proportional?

Yes, the relationship is proportional because the constant is the same for all values in the table.

2b.) How many plants would 8 seed packets grow?

$y = 8(10) = 80$ plants

2c.) How many seed packets would make 90 plants?

$\frac{90}{10} = \frac{10x}{10}$ $x = 9$ seed packets

How to Determine if a Relationship is Proportional from a GRAPH:

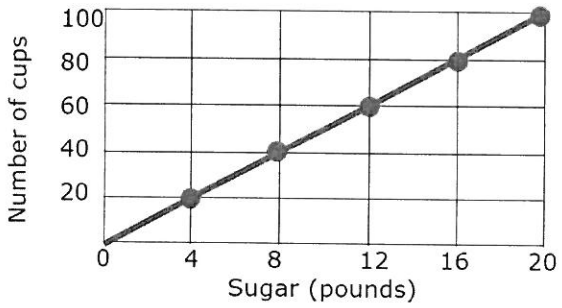
1.) If the graph makes a straight line, then the relationship IS proportional

2.) If the graph does NOT makes a straight line, then the relationship is NOT proportional

Examples

Determine if each graph makes a proportional relationship. If it does, find the constant of proportionality.

3.) The graph below represents the total number of cups of coffee and the total amount of pounds of sugar to make the coffee.



3a.) Is the relationship proportional?

Yes, the relationship is proportional because it makes a straight line.

3b.) What is the constant of proportionality?

$\frac{20}{4} = 5$ lbs/cup

3c.) Write an equation to represent the relationship.

$y = 5x$

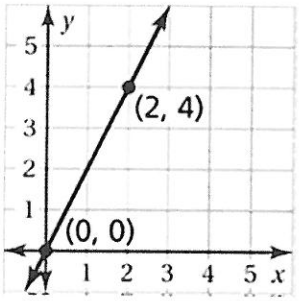
3d.) Find the number of cups of coffee you can make with 10 pounds of sugar.

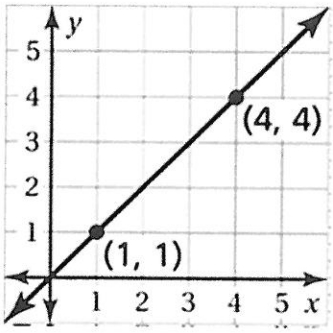
$y = 5(10) = 50$ cups

5.6 Direct Variation Teacher Notes

POD:

1.) Find the slope of the line = $\frac{2}{1}$ or 2 2.) Find the slope of the line = $\frac{1}{1}$ or 1





Vocabulary:

Direct Variation: When y = *k*x (also known as y = mx). *K* is a number and it cannot equal zero.
Example of Direct Variation: y = 3x or y = $\frac{1}{2}$ x

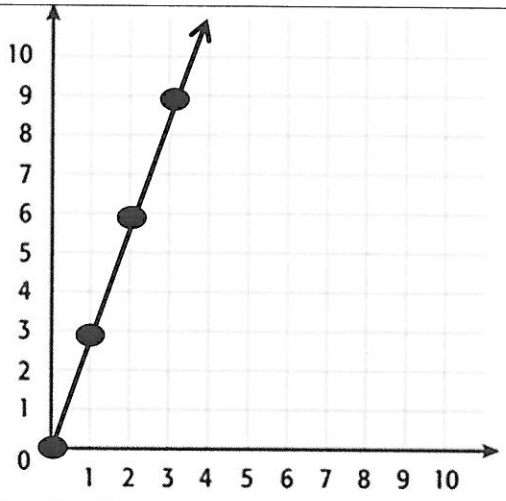
Constant of Proportionality: *k* is the constant of proportionality. When you graph the equation y = *k*x, the points must form a straight line AND the line MUST pass through the point of origin in order for x and y to be in DIRECT VARIATION.

1a.) Plot the points:

x	0	1	2	3
y	0	3	6	9

1b.) Does the graph show direct variation?
Yes, it passes (0,0) and is a straight line.

1c.) Write the equation:
y = 3x



Tell whether x and y show direct variation (remember: $y = kx$)

2.) $y + 1 = 2x$ (SOLVE FOR Y first)

$-1 \quad -1$

$y = 2x - 1$

Is this direct variation? Yes or No

Why? Because the equation cannot be written as $y = kx$

3.) $\frac{1}{2}y = x$ (SOLVE for y first)

Multiply both sides by the reciprocal.

$y = 2x$

Is this direct variation? Yes or No

Why? Because the equation CAN be written as $y = kx$

4.)

x	1	2	3	4
y	4	8	12	16

Yes, the line passes through (0, 0) and points are in a straight line.

5.)

x	-2	-1	0	1
y	4	2	0	2

No, the points do not lie on a line.

The variables *x* and *y* vary directly. Use the values to find the constant of proportionality and write an equation that relates *x* and *y*.

6.) $y = 6; x = 3$

$\frac{6}{3} = 2$

$y = 2x$

7.) $y = 4; x = 6$

$\frac{4}{6} = \frac{2}{3}$

$y = \frac{2}{3}x$